Math 250 – Sect 3.9– Linear Approximations and Differentials

I. Linear Approximations:
One way to define *differentiability* at a point is that the function must be continuous at the point, and the derivative must exists.

Pictures:

ANOTHER way to define differentiability is to say that the function must have *local linearity.* The TANGENT LINE at a point can also be defined as a **linear approximation** to the function.

This tangent line (or *linear approximation*) can be used to approximate values for the function near the point of tangency.

-example- Find the linear approximation to $f(x)=\sqrt{2x+1}$ at $x = 4$, then use it to approximate the value of $f(4.2)$.

-example- Find the linear approximation for $y=\sin x$ at $x = 0$, and use it to approximate $\sin(0.3)$.

II. **Differentials**

Recall, the DERIVATIVE values represent SLOPE values on a given function.

Slope = 		Derivative =

The quantities, *dy* and *dx* are called "differentials", and represent small, nonzero values for the changes in *y* and *x*.

Specifically, if $y = f(x)$, then

Differentials are used to *approximate* changes in the output values of a function given small changes in the input values. THIS IS REALLY JUST THE SAME AS UTILIZING LINEAR APPROXIMATIONS, but allows us to shortcut the process of writing the equation of the line.

Picture:

-example- Find *dy*.

a. $y = 4x^3 + 7x - 3$

b. $y = \sqrt{5 - x^2}$

Math 250 – Sect 3.9– Linear Approximations and Differentials

-example- Consider the function $y = \sqrt{x}$.

a. Find the value of dy given that $x = 1$ and $dx = 0.1$

b. Find the value of dy given that $x = 9$ and $dx = -0.2$

c. Use the information from (b) to approximate the value of $\sqrt{8.8}$

-example- The radius of a circle is measured to be 4.8 cm, with a possible error of ±0.1 cm. Use differentials to find the possible error in the measurement of the:

a. Circumference

b. Area

c. What is the *percent error* in the area measurement?

d. If the radius is measured to be 4.8, what is the maximum allowable error in the measurement if the error in computing the area cannot exceed 2%?

-example- A machine is designed to manufacture ball bearings with a radius of 0.8 inches, ± 0.05 inch. Use differentials to estimate the range of volumes possible for a ball bearing manufactured by this machine.

REVIEW (separate paper):

1. Consider the function $f(x) = \dfrac{x}{x+2}$.

a. Determine any vertical or horizontal asymptotes for *f*.

b. Determine the interval(s) on which *f* is increasing/decreasing, and the locations of any maximums/minimums.

c. Determine the interval(s) on which *f* is concave up/ concave down, and the locations of any points of inflection.

2. The sum of the perimeters of a circle and a square is 16. Find the dimensions of the circle and square that produce a minimum total area.

3. Use Newton's Method with an initial guess of $x = 2$ to find the zero for the function $y = \dfrac{1}{3}x^3 + x - 6$.

4. Use differentials to approximate the value of $\sqrt[3]{29}$.